On the Performance of the Representative Agent During Off-Equilibrium Dynamics*

Zakaria Babutsidze

UNU-MERIT
School of Business and Economics
Maastricht University

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Abstract

This essay contributes to the discussion about the representative agent's ability to characterize the collection of agents. We perform two exercises in context of a fairly general multi-product multi-agent environment and show that in order the representative agent to be able to describe the off-equilibrium dynamics of the society one needs additional assumptions. It is established that this feature is not specific to the economies with interacting agents.

1 Introduction

Whether the behaviour of one individual can accurately describe the behaviour of the collection of agents is the topic of decades-long debate in economics. Some people believe the approximation is useful, even if not very accurate, while others think approximation is fundamentally misleading. This central player, on which the bulk of the discipline is built, is called the "representative agent."

This essay aims to contribute to the debate by demonstrating that the representative agent can well describe the economic system only in special cases. Most of the previous

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effort has gone into the analysis of the representative agent's powers in equilibrium (Kirman, 1989; 1992). Here we are concerned with her ability to describe the off-equilibrium behaviour of dynamic economy. This essay, to some extent, motivates the remaining of the thesis methodologically by demonstrating that the representative agent models can not be used for studying the economies that are under discussion in the remaining of the thesis. Thus the use of heterogenous models is justified.

To analyze the problem we set up a very simple behavioral model and study its off-equilibrium dynamics. The important feature of the model is that consumers are learning while the model is out of equilibrium. Learning is the force that drives the economy to time invariant state, that we use as the definition of equilibrium. The equilibrium in itself is not very interesting. It is characterized by consumer homogeneity, thus we can predict without any mathematical analysis that in this state the representative agent will be powerful.

In our simple model consumers do only one thing upgrade their consumption skills. We consider two variants of the model. One, where consumers are learning from own consumption history in a multi-product environment. This is the case where consumers are heterogenous but do not interact with each other. In the other variant of the model consumers are interacting with each other and sharing their skills. Here heterogenous consumers interact with each other. We demonstrate that in both cases one needs certain distributional requirements in order the representative agent to be able to describe the dynamics of the society off-equilibrium.

The remaining of the essay is organized as follows. Section 2 reviews the literature on the debate about the performance of the representative agent. Section 3 presents the setup of the economy for the exercises. Section 4 presents the results. The last section concludes.

2 The Representative Agent and Its Problems

The predecessor of the modern representative agent is, without a doubt, Marshall's (1920/1961) "representative firm." Marshall used this notion in order to speculate about the industry-level supply curve. He recognized that firms in the industry might have differing costs. This created a problems as it brought a confusion into which of those costs was the one determining the unique selling price in the industry. Marshall invented the notion of the representative firm in order to resolve this problem. By his definition it was a "fairly" successful firm that was managed with "normal" ability. In principle Marshall never had in mind the representative firm to be some kind of a statistical construct.

Marshall's notion of the representative firm was met with the fierce criticism from colleagues (e.g. Robbins, 1928) and it was ultimately abandoned. However the idea kept on living and it found revival as the central player of the modern neoclassical economics. But nowadays it is somewhat different from Marshall's original idea. Today the representative agent is a statistical construct. It is an "average' agent of the economy. Although we were not able to pin down an exact definition of the modern representative agent anywhere in the literature, after the analysis of numerous uses of the concept we feel confident to state that the representative agent is the construct that describes the "average' values of agent-specific variable distributions.

Although the definition of the representative agent in the previous paragraph seems fairly clear there are two problems with it. Firstly, what does the "average' exactly mean? There are two different answers to this question in the literature: the simple average of the distribution (mean) (e.g. Schlee, 2001) or the average weighted with some other characteristic[1] (e.g. Constantinides and Duffie, 1996). In principle there is a third one, which is to use the sum rather than the average (e.g. Gollier and Zeckhauser, 2005), but this is equivalent to the simple average. Although having two definitions of the average does not create the conceptual problem (after all you use the one most relevant in your case) it does create the problem of comparability of results across researches.

Even if one ignores the problem of *which average*, more important problem remains. This problem is: the average of *which variable* is the representative agent supposed to describe? This is an acute problem especially in models where an agent has more than one function. Geweke (1985) constructs the model where firms are playing three roles: they produce, they demand production factors and supply the products. He demonstrates that in this economy there will be three different representative agents: the average producer, the average supplier and the firm placing average demand on production factors. In

[1] For example income in case of consumers or size in case of firms.

all three cases representatives are different from each other. Modelers working with the representative agent somehow choose one of the definitions (that is hardly ever presented in the paper), which again creates the problem of compatibility.

Besides problems with the definition of the representative agent the decades-long literature has brought up three major topics of the discussion. Firstly, whether the representative agent can be constructed at all? In this respect the fundamental contribution is due Rubinstein (1974) who provided fairly general sufficient conditions under which representative agent can be constructed. But this goes back to the definition of the representative agent. As Kirman (1992) argues there can be cases where these kind of individuals will not accurately represent the society. In particular one can construct the situation when this kind of representative agent prefers the first option over the second, while *every* member of the society prefers the second over the first. Then we have an issue of whether the representative agents that can be constructed are useful.

Secondly, imagine we construct the agent that describes the behaviour of the average of some variable well. It might well be that behavioral rules of this agent are different from the the behavioral rules of the society that it represents. Caballero (1992) points out that the representative agent framework has "blured the distinction between statements that are valid at the individual level and those that apply to the aggregate." Schlee (2001) calls this the problem of "normative representativeness" of the representative agent. Maliar and Maliar (2003) present the recent example of the economy where the representative agent has different behavioral rule then every other agent in the economy (that are homogenous in that respect). Researchers feel comfortable working with these kinds of models (e.g. Gollier and Zeckhauser, 2005) as the ultimate goal of describing the average behaviour is accomplished. However, there is a fundamental problem with this approach which makes it redundant. If every agent in the economy has one behavioral rule and the representative agent (able to describe the average behavior) has a different behavioral rule this rule has to be constructed. In order to construct the behavioral mechanism of the representative agent one has to solve the model without the representative agent. If the solution without the representative agent is obtained there is no use in further reformulation of the problem as the problem with the representative agent. Thus, we believe that the only useful representative agent has to be also "nominal' representative of the society.

Third, relatively minor, problem with the representative agent framework is that when using it one can not address distributional issues (Stoker, 1986). As the representative agent is a single individual distributions of the characteristics over the population can not be observed and analysed. More recently Caselli and Ventura (2000) proposed the methodology for constructing the representative agent that will be able to describe the distribution of characteristics in the society. However, again, the agent that they use is not a nominal representative of the society. We think these kinds of framework are useful, however they have to be called something like the single-agent (rather than representative agent) economies.

All in all, together with certain useful things, the representative agent framework has brought several acute problems in economics. Most importantly the issue of its meaning (exact definition) and consequently of the compatibility of results across researches.

3 Framework

As discussed in the previous section if you construct your representative agent to be a nominal representative of your society she might not be able to properly describe the economy in equilibrium. If you construct the representative agent to describe the economy in equilibrium it might not be a nominal representative of the economy. Of course there are instances when the nominal representative agent can describe the economy properly. Two such examples are presented in Huffman (1986) and Salyer (1988). Then it becomes important to understand when this is the case. If one demonstrates that the problem at hand qualifies in this category there will be no further objections for using representative agent methodology.

However, there is a substantial body of literature demonstrating that one can not obtain certain empirically validated features with representative agent frameworks. Examples of these kind of work are Grossman and Shiller (1981) who find that the rate of risk aversion implied by the representative agent model are implausibly high, and Mehra and Prescott (1985) who obtain implausibly high risk premium. Several studies contrasting representative and heterogenous agent models have confidently rejected representative agent models (e.g. Hansen and Singelton, 1983; Cooper and Haltiwanger, 1990).

All the works referred in this essay are concerned with the ability of the representative agent to describe the equilibrium properties of the economy. There is no contribution analysing the off-equilibrium performance of the representative agent. This seems reasonable as the equilibrium is of the central importance in modern economics. However, in recent years there has been substantial increase in research into off-equilibrium behavior. In fact in all the remaining essays of this thesis off-equilibrium dynamics plays a crucial role. Thus for the choice of the methodology it is important to evaluate the performance of the representative agent in types of environments that will be discussed in the remaining of the paper.

Closest to this essay comes an exercise by Geweke (1985) who evaluates the performance of the representative agent in evaluating the effect of the policy change. This is studying the adjustment process to the new policy, which is similar to the analysis of the off-equilibrium dynamics, as the original arrangement is not equilibrium after the policy change. Geweke (1985) constracts three different types of representative agents and demonstrates that none of the three can predict the average effect of the policy change correctly.

What we do in this essay is somewhat similar: we start our economy from the point off-equilibrium and study the transition to the equilibrium. But we analyse simpler and more general setup. There are no multiple role playing agents in our economy so that we need several definitions of the representative agent. Our agents do in principle only one simple task: learn. There are two main features (besides consumer heterogeneity) uniting essays in this thesis. These are learning and interaction. We analyse these two features in turn: first learning without interaction, next learning through interaction.

The setup of the economy that we use in this essay is extremely simple. There are large number of agents. Think of them as consumers that have to make decision about which product to consume in a multy-product environment. Each consumer has a certain skill level for each product. Besides these skill levels consumers are absolutely identical (e.g. no difference in income) so that we do not go into the issues of simple versus weighted average.

Laws of motion of skill levels are identical for every consumer. The skill levels are positive, non-decreasing and bounded from above. If we define the equilibrium as the time invariant state of the economy, all equilibria will be characterized by the homogeneity of agents. Of course, in this case the representative agent will have no problem describing the equilibrium. However, our concern is with the off-equilibrium dynamics.

In this environment we can define the representative agent.

Definition 1 *The representative agent is an agent that behaves identically to every other agent in the economy and describes the evolution of the average skill level from the initial state to the equilibrium.*

From the definition above it is obvious that we have in mind the "nominal' representative agent. As argued earlier, construction of any other type of representative agent would not make much sense. If the representative agent, as described in definition 1, exists it can be further used for the analysis. If it does not exist, in order to construct another representative agent (that would clearly have difference behavioral rules from every consumer in this economy) we have to first solve for the evolution path of the average skill level and only after that create the representative agent that would mimic it. Even if one ignores the possibility of many behavioral rules being able to replicate the exact same route of average skill evolution, the use of such kind of the representative agent will be pretty limited: analysis of a slightly different problem might call for the different representative of the economy.

Next we analyse two problems of skill evolution in turn and evaluate the performance of the representative agent as defined. More precisely we ask the question whether the representative agent can be constructed.

4 Results

4.1 Learning by Consuming

Consider the situation where in order consumers to acquire consumption skills they have to consume products. This construct is similar to the notion of learning to consume (Witt, 2001). With time as agents are consuming products they learn and ultimately (as $t \to \infty$) everybody's skill level for every product converges to the maximum. To simplify the presentation assume that there are several products on the market for already long enough so that every agent already has maximum skill levels for all of them. Then the economy is in equilibrium and all the product choices are time invariant.

Now consider the new product entering the market. At time zero consumer skills for the new product will be distributed over the population. This distribution is described by function $F_0(s)$. Assume, reasonably, that consumer skills are increasing (at a decreasing rate, although this is not crucial for the results) in number of times consumer has used the product. Without loss of generality we assume the skill levels are bounded by unity. Then we can write down the law of motion of the skill level for the new product for agent i

$$s_n^i = 1 - \left(1 - s^i(0)\right) e^{-\delta n}, \tag{1}$$

where s is a skill level, n is how many times an agent i has consumed this particular product, δ is the speed of learning and $s^i(0)$ is the initial skill level.

From equation (1) we can derive the change in skill levels between two consumptions

$$s_{n+1}^i - s_n^i = \gamma(1 - s_n^i), \tag{2}$$

where $\gamma = 1 - e^{-\delta}$.

The skill level dynamics for each of the agents depend on her product choices. In order to derive the expected path for the skills, assume $P(s)$ is the probability that the consumer will choose an entrant product if her skill level is s. $P(\cdot)$ is continuous increasing function: the more skills an agent has for the new product, the higher the chance that she will purchase it, *ceteris paribus*. Naturally, probabilities for every agent add to one across all the products at every time period.

For studying the skill level development we need the expected law of motion for the skill of every agent. The agent i with the skill level $s^i(t)$ consumes the product at time t with the probability $P\left(s^i(t)\right)$. Therefore, with the same probability agent i's skills increase by $\gamma(1 - s^i(t))$, but with the probability $1 - P\left(s^i(t)\right)$ they remain at the same level. With this logic, we can write down the expected law of motion:

$$s^i(t+1) = s^i(t) + \gamma \left(1 - s^i(t)\right) P\left(s^i(t)\right). \tag{3}$$

Starting from the initial distribution, using equation (3), we can calculate the expected path of the average skill for the new product in the economy.

Notice that the equation (3) is agent-specific and if one assumes that the initial distribution is not given by Dirac's delta function, we will have heterogeneity in skill development paths across population. In this context the question arises: do we need to track the skill level of every agent or one can construct the representative agent which can describe the dynamics of the society. According to the definition, the representative agent has to have the (expected) average skill level in the economy if she wants to claim being a representative.

Thus, we know that our representative agent has the average skill level in the initial skill distribution: $s^r(0) = \bar{s}(0)$. Then, her skill level next period should be

$$s^r(1) = \bar{s}(0) + \gamma \left(1 - \bar{s}(0)\right) P(\bar{s}(0)). \tag{4}$$

The average skill level in the economy at time one is

$$\bar{s}(1) = \int s dF_1(s) = \int \left(s + \gamma(1-s)P(s)\right) dF_0(s), \tag{5}$$

which can be rewritten as

$$\bar{s}(1) = \bar{s}(0) + \gamma \int (1-s)P(s)dF_0(s). \tag{6}$$

From equations (4) and (6) it is obvious that for $s^r(1) = \bar{s}(1)$ to hold we need

$$\left(1 - \bar{s}(0)\right) P\left(\bar{s}(0)\right) = \int (1-s)P(s)dF_0(s). \tag{7}$$

Equality (7) is the requirement for our representative agent at time zero to be representative at time one. But the representative agent has to be able to describe the average skill level in the economy at every time period. Thus we need the following general equality to hold

$$(1 - \bar{s}(t))\, P\left(\bar{s}(t)\right) = \int (1-s)P(s)dF_t(s). \tag{8}$$

Now we have all the material in order to be able to answer the question: can the representative agent be constructed in this economy? It can only in exceptional cases, three of which are presented here.

Examples: There are three straightforward examples that one can construct when the representative consumer will describe the economy precisely. First of them is when γ is zero. This is easy to infer from equation (4). However, this case means that there is no learning in the economy, thus no dynamics of skill levels. Consequently the economy is already at the equilibrium. And by construction the representative agent is perfectly able to describe the economy. Another example is when $\bar{s}(0) = 1$, which again implies that the economy starts off at the equilibrium. An example involving learning is the case when the function $P(\cdot)$ is constant. However, this case completely undermines the model's central assumption that the choice of consumers is inferred from the skill levels for the products. Here, although there will be skill level dynamics, will be no dynamics in purchases. Thus, the skill levels themselves become non-interesting variables.

Any other instance where the representative agent will exist in the economy will involve joint restrictions on functions $P(\cdot)$ and $F_t(\cdot)$, $\forall t$. Without these restrictions the representative agent at one time period will not be representative of the economy in consecutive periods. Thus, in general, the representative agent that can describe the off-equilibrium dynamics of this simple economy can not be constructed.

4.2 Skill Sharing

Now consider the case when there is an interaction among agents. We do not assume anymore that consumers learn by consuming. Rather, we assume that they learn by interaction with each other. In this environment multiplicity of products does not play a role, thus we consider there is only one product in the economy. Skill distribution over population at time zero is again described by $F_0(s)$. Interaction structure is as follows. Every period, every agent (i) randomly picks one other agent (j) from the population. If

j has higher skill level then i, i learns from j. As a result her skills increase by $\mu(s^j - s^i)$. If j's skill level is lower than that of i's, i can not learn anything.

There are several things to note in this scheme. The agent who has the highest skill level in initial distribution (s'), can not learn anything from anybody in the population. Everybody else's skill level approaches her skill level as $t \to \infty$. Here again, we are interested in off-equilibrium dynamics, or whether the representative agent can predict how the average skill level approaches its time-invariant value.

Given the description of the model we can specify the law of motion for skill level of an agent i

$$s^i(t+1) = s^i(t) + \mu \int_{s^i(t)}^{s'} f_t(s)(s - s^i(t))ds, \tag{9}$$

where $f_t(s)$ is the probability density function of skill distribution at time t. Thus, the the second summand in the right hand side gives the expected increase in the skill level.

Our representative agent at time zero has to have a skill level equal to the average of the society

$$s^r(0) = \bar{s}(0) = \int s dF_0(s). \tag{10}$$

Then, her skill level at time one will be

$$s^r(1) = \bar{s}(0) + \mu \int_{\bar{s}(0)}^{s'} f_0(s)(s - \bar{s}(0))ds. \tag{11}$$

We can also calculate the average skill level in the economy at time one

$$\bar{s}(1) = \int s dF_1(s) = \int \left(s^i(0) + \mu \int_{s^i(0)}^{s'} f_0(s) \left(s - s^i(0)\right) ds \right) dF_0(s^i(0)) \tag{12}$$

that can rewritten as

$$\bar{s}(1) = \bar{s}(0) + \mu \int \int_{s^i(0)}^{s'} f_0(s) \left(s - s^i(0)\right) ds dF_0(s^i(0)). \tag{13}$$

By looking at equations (11) and (13) for the equality $s^r(1) = \bar{s}(1)$ to hold we need

$$\int_{\bar{s}(0)}^{s'} f_0(s)\,(s - \bar{s}(0))ds = \int \int_{s^i(0)}^{s'} f_0(s)\left(s - s^i(0)\right) ds dF_0(s^i(0)). \tag{14}$$

Or, in general

$$\int_{\bar{s}(t)}^{s'} f_t(s)(s - \bar{s}(t))ds = \int \int_{s^i(t)}^{s'} f_t(s)\left(s - s^i(t)\right) ds dF_t(s^i(t)). \tag{15}$$

Now, again, we have all the material to be able to answer whether the representative agent can be created in this economy. And, again, it can be only in special cases.

Examples. Examples of the cases when the representative agent will be able to describe the economy accurately are when $\mu = 0$ or when $s^j(0) = s^r(0)$, $\forall j$. Both of these cases, just like examples in section 4.1, imply that the economy starts off at the equilibrium, thus we can not evaluate the performance of the representative agent off-equilibrium.

Any other case in which the representative agent of the economy can be constructed would require restrictions on the skill distribution functions $F_t(\cdot)$, $\forall t$. Thus, again, in general the representative agent that could describe the off-equilibrium dynamics of the simple economy can not be constructed.

5 Conclusion

In this essay we have discussed two types of behavior of collection of heterogeneous agents. We have shown that even if the representative agent is powerful in equilibrium she might not be able to describe the off-equilibrium dynamics of the society. In particular, we have shown that the representative agent at time t will not be representative at time $t + 1$ if she follows the same behavioral rules as every other agent does. By analysing two setups, one with interaction, another without, we have also established that this phenomenon is not specific to models with interaction.

The definition of the representative agent that we have used is the "nominal representative" of the society. Which effectively means that if all the agents follow one behavioral rule, the representative has to also follow the same behavioral protocol. Of course, in this simple economies one can construct the the agent which will mimic the dynamics of averages of interesting variables in the economy, but as we demonstrated, this kind of agent will have different behavior from every agent in the economy. Besides unfairness of the title "representative" in this case, there are severe problems with this kind of central agents.

Firstly, in order to construct them one has to solve the heterogenous agent model completely. For example in our case, we had to obtain the expected path of the average skill level in the economy before thinking of constructing the representative agent that could describe it. Thus, the usefulness of this kind of an agent is questionable. Secondly, even if we have the full solution there can be multiple protocols that can replicate the solution. As the behaviour of actual agents is not the restriction for the behaviour of the representative we are confronted with the choice problem. Of course the choice can be aided by some other criteria, for example there features of the economy that we want to describe with our representative agent. However, in this case again in order to make a selection we have to have the full solution of that part of the model too. Adding dimensions to the model might narrow down the list of representative agents, but it definitely does not make choice easier. And finally, even if we somehow make a choice, we can not use the chosen representative agent to analyse any aspect of the economy that has not been included in the choice criteria (thus has not been solved for). We simply can not be sure that the representative agent is the representative for that part of the model.

References

Caballero, R. J. (1992): "A Fallacy of Composition," *The American Economic Review*, 82(5), 1279–1292.

Caselli, F., and J. Ventura (2000): "A Representative Consumer Theory of Distribution," *The American Economic Review*, 90(4), 909–926.

Constantinides, G. M., and D. Duffie (1996): "Asset Pricing with Heterogeneous Consumers," *The Journal of Political Economy*, 104(2), 219–240.

Cooper, R., and J. Haltiwanger (1990): "Inventories and the Propagation of Sectoral Shocks," *The American Economic Review*, 80(1), 170–190.

Geweke, J. (1985): "Macroeconometric Modeling and the Theory of the Representative Agent," *The American Economic Review*, 75(2), 206–210.

Gollier, C., and R. Zeckhauser (2005): "Aggregation of Heterogeneous Time Preferences," *The Journal of Political Economy*, 113(4), 878–896.

Grossman, S. F., and R. J. Shiller (1981): "The Determinants of the Variability of Stock Market Prices," *The American Economic Review*, 71, 222–227.

Hansen, L. P., and K. J. Singelton (1983): "Stochastic Consumption, Risk Aversion and the Temporal Beravior of Asset Prices," *The Journal of Political Economy*, 91, 249–265.

Huffman, G. W. (1986): "The Representative Agent, Overlapping Generations, and Asset Pricing," *The Canadian Journal of Economics / Revue canadienne d'Economique*, 19(3), 511–521.

Kirman, A. (1989): "The Intrinsic Limits of Modern Economic Theory: The Emperor has No Clothes," *The Economic Journal*, 99(395), 126–139.

Kirman, A. P. (1992): "Whom or What Does the Representative Individual Represent?," *The Journal of Economic Perspectives*, 6(2), 117–136.

Maliar, L., and S. Maliar (2003): "The Representative Consumer in the Neoclassical Growth Model with Idiosynchratic Shocks," *Review of Economic Dynamics*, 6, 362–380.

Marshall, A. (1920/1961): *Principles of Economics*, vol. 9th variorum. Macmillan and Co., London.

Mehra, R., and E. C. Prescott (1985): "Equity Premium: A Puzzle," *The Journal of Monetary Economics*, 15, 145–161.

Robbins, L. (1928): "The Representative Firm," *The Economic Journal*, 38, 387–404.

Rubinstein, M. (1974): "An Aggregation Theorem for Securities Markets," *Journal of Financial Economies*, 1, 225–244.

Salyer, K. D. (1988): "Overlapping Generations and Representative Agent Models of the Equity Premia: Implications from a Growing Economy," *The Canadian Journal of Economics / Revue canadienne d'Economique*, 21(3), 565–578.

Schlee, E. E. (2001): "The Value of Information in Efficient Risk-Sharing Arrangements," *The American Economic Review*, 91(3), 509–524.

Stoker, T. M. (1986): "Simple Tests of Distributional Effects on Macroeconomic Equations," *The Journal of Political Economy*, 94(4), 763–795.

Witt, U. (2001): "Learning to Consume - A Theory of Wants and the Growth of Demand," *Journal of Evolutionary Economics*, 11, 23–36.